Conservation theorems & Symmetry

2 Concept of Symmetry →

If any system or any function representing a proper property of the system does not change under some operation carried on the system, the system is said to possess a symmetry with respect to that operation.

$\partial L / \partial t = 0$

Langrangian of a closed system should be invariant under the operations of translation and rotation in space and that it should not depend explicitly on time.

(1) Homogeneity of time & Conservation of energy →

If for any arbitrary displacement of the origin of time, the physical properties of any closed system remain unaffected, then time is homogeneous.

For a closed system $\partial L / \partial t = 0$ and there is no external force, K.E & P.E. are constant

$$E = T + V = \text{constant}$$

Hence, due to homogeneity of time, total energy is conserved for any closed system.

(2) Homogeneity of space and Conservation of linear momentum

If for any arbitrary displacement of the origin of any reference fram of the physical propeties of all closed systems remain unaffected the space is homogeneous.

The Lagrangian of a closed system should not change due to any arbitrarity small uniform ~~translat~~ translation of all particles.

As for closed system external force is zero then $-Q_J = 0$

$$\dot{P}_i = 0$$

$$P_i = \text{constant}$$

The total linear momentum of any closed system is conserved due to the homogeneity of space

(2) Isotropy of space

If for any arbitary rotation about the origin of any reference frame the physial propets of any closed, system remain unaffected the space is called isotropic.

The Lagrangian for a closed system should not change due to any arbitrary small rotation of referene frame about arbitrary direction. For a closed system external torque is zero

$$Q_J = \sum_i F_i \cdot \frac{\partial r_i}{\partial q_J} \quad \text{Force}$$

$$\frac{\partial r_i}{\partial q_J} = \frac{\partial r_i}{\partial \theta} = n \times r_i \quad (n = \text{unit vector})$$

r_i (notation of the vector)

$$Q_J = \sum_i F_i \cdot n \times r_i$$

$$= \sum_i n \cdot r_i \times F_i$$

$$= n \sum_i r_i \times F_i = n \sum_i \underbrace{r_i \times F_i}_{N_i \text{ torque}}$$

$$= n \cdot N$$

$$P_J = \frac{\partial T}{\partial \dot{q}_J} = \sum_i m_i v_i \cdot \frac{\partial r_i}{\partial q_J}$$

$$= \sum_i n \cdot r_i \times m_i v_i = n \cdot \sum_i r_i \times m_i v_i$$

$$= n \cdot L$$

Thus the rotation coordinate q_J is cyclic then Q_J which is component of applied ~~component~~ torque along n vanishes and the component of L along n is constant Hence the angular momentum conservation theorem [illegible] of the general Conservation theorem relating to cyclic coordinate